Axial Light Field for Curved Mirrors: Reflect Your Perspective, Widen Your View

Yuichi Taguchi, Amit Agrawal, Srikumar Ramalingam, and Ashok Veeraraghavan
Mitsubishi Electric Research Labs (MERL), Cambridge, MA, USA
{taguchi, agrawal, ramalingam, veerarag} at merl.com

Supplementary Material

Here we analyze the number of light rays in a virtual light field that are captured by a real camera in a planar/axial light field by deriving equations for sphere. As described in Section 3.1, such light rays lie on a plane of reflection, defined by the rotation axis of a mirror (the sphere center in this case) and the real and virtual viewpoints. We therefore consider the problem on this reflection plane.

For sphere, we can always define a coordinate system where the origin is the sphere center and a line connecting the sphere center and the virtual viewpoint corresponds to an axis, as shown in Figure 1. Our goal is to find solutions for the angle ϕ that satisfy geometric constraints of reflection of rays, given the positions of the real and virtual viewpoints. Below we first describe a case where the real viewpoint lies on the axis, referred to as axial case, and then explain a general case where the real viewpoint lies away from the axis, referred to as non-axial case.

Axial case: As shown in Figure 1, a ray originating from the virtual viewpoint at $(0, h)$ intersects the sphere at $(r\cos\phi, r\sin\phi)$. After reflection, the ray passes through the real viewpoint at $(0, q)$, which gives the following constraint:

$$q - r\sin\phi = -r\cos\phi\tan(2\phi - \alpha). \tag{1}$$

Here, α is given by

$$\tan\alpha = \frac{r\sin\phi - h}{r\cos\phi}. \tag{2}$$

Using $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ and solving the equation for ϕ gives

$$\sin\phi = 1 \tag{3}$$

$$\sin\phi = \frac{r(h+q)}{2hq}. \tag{4}$$

The first solution that directly connects the real and virtual viewpoints is a trivial one, which always provides one ray.

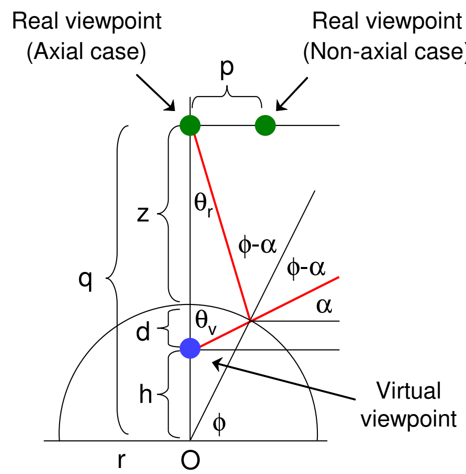

Figure 1. Configuration in a plane of reflection. We define $h = r - d$ and $q = r + z$ for simplicity.

Depending on the positions of the real and virtual viewpoints, the second solution provides two rays corresponding to $\phi_0 = \arcsin(\frac{r(h+q)}{2hq})$ and $\pi - \phi_0$, or no rays when $h < \frac{rq}{2q-r}$. Hence, there is a maximum of three rays on this plane. The position $h = \frac{rq}{2q-r}$ corresponds to the cusp [1] of the caustic defined by the real viewpoint. Note that since there is an infinite family of reflection planes in the axial case, the real cameras in an axial light field capture a cone of rays when each plane contains three rays.

Non-axial case: Next we describe the case where the real viewpoint does not lie on the axis connecting the sphere center and the virtual viewpoint. In this case, the reflected ray passes through the real viewpoint at (p, q), which changes (1) to

$$q - r\sin\phi = (p - r\cos\phi)\tan(2\phi - \alpha). \tag{5}$$

By defining $\cos\phi = t$ and $\sin\phi = \sqrt{1-t^2}$ and expanding the equation, we have the following fourth-degree equation

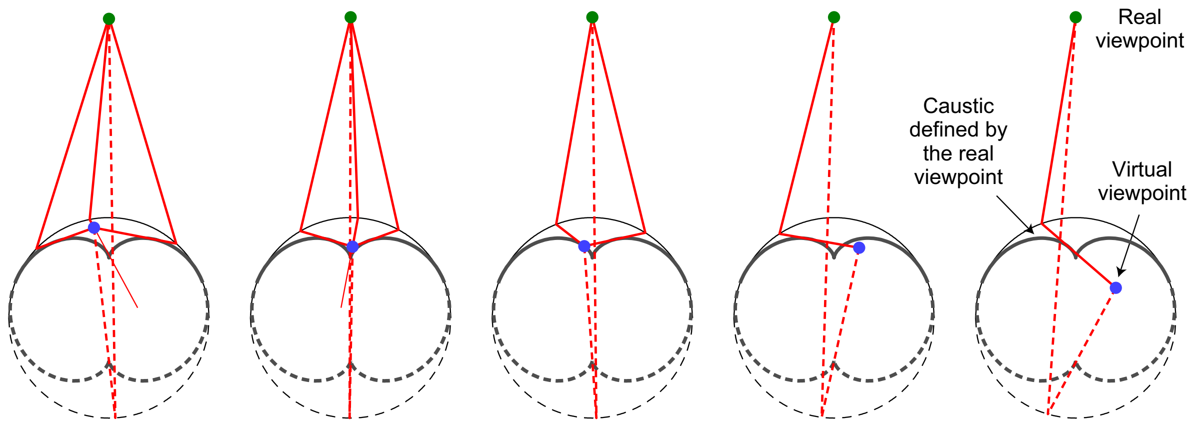

Figure 2. Computed rays for different positions of the real and virtual viewpoints. Infeasible rays that cannot be captured are depicted as dashed lines. (a, b) The number of actual rays is three when the virtual viewpoint is above the caustic defined by the real viewpoint. (c) It becomes two for the virtual viewpoint on the caustic and (d, e) one below the caustic. In (d, e), the other two solutions are complex numbers. Here the sphere center and the real viewpoint are aligned perpendicular in order to show the relationship between the caustic and the virtual viewpoint/reflected rays.

about t:

$$At^4 + Bt^3 + Ct^2 + Dt + E = 0 \tag{6}$$
$$\begin{aligned} A &= 4h^2(p^2 + q^2) \\ B &= -4prh^2 \\ C &= -4h^2(p^2 + q^2) + r^2(p^2 + (q + h)^2) \\ D &= 2prh(h - q) \\ E &= p^2(h^2 - r^2) \end{aligned}$$

The number of solutions of the above equation depends on the positions of the real and virtual viewpoints with respect to the sphere and could be a maximum of four. Figure 2 shows the rays computed from the solutions for several different real and virtual viewpoints. We observed that when the virtual viewpoint is above the caustic defined by the real viewpoint, the number of solutions is four, in which three solutions correspond to actual rays and the other one corresponds to an infeasible ray, which has a reflection point on the bottom of the sphere. The number of actual rays becomes two for the virtual viewpoint placed on the caustic and one below the caustic. Since there is only a single plane of reflection in the non-axial case, the real cameras that are not on the mirror axis in a planar light field capture a maximum of three rays in practice.

References

[1] R. Swaminathan, M. Grossberg, and S. Nayar. Non-single viewpoint catadioptric cameras: Geometry and analysis. *Int'l J. Computer Vision*, 66(3):211–229, Mar 2006. 1